

Magnetic properties of superconducting multifilamentary tapes in perpendicular field.

II: Horizontal and matrix arrays

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Current and field profiles, and magnetization and ac losses are calculated for arrays of infinitely long superconducting tapes in the critical state in a perpendicularly applied magnetic field. The tapes are arranged both horizontally and in a matrix configuration, which is the geometry found in many actual high- T_c superconducting tapes. The finite thickness of the tapes and the effects of demagnetizing fields are considered. Systematic results for the magnetization and ac losses of the tapes are obtained as function of the geometry and separation of the constituent tapes. Results allow to understand some unexplained features observed in experiments, as well as to propose some future directions.

I. INTRODUCTION

High temperature superconducting (HTSC) cables have a large potential for many applications where a very high current intensities are needed, such as power transmission cables, magnets, superconducting magnetic energy storage systems (SMES), transformers, and motors [1,2]. In particular, silver sheathed $\text{Bi}_2\text{Sr}_2\text{Ca}_2\text{Cu}_3\text{O}_{10}$ (Ag/Bi-2223) tape conductors showed to be the HTSC most used for practical devices, due to the good superconductor material quality and the feasibility to make kilometer long cables. Many of the HTSC cables applications work under AC conditions, like power transmission cables, transformers and motors. An important problem for the superconducting power devices operating at AC intensities is caused by their power losses [3], which must be reduced as low as possible to justify the expenses of the superconducting material and the cryogenic system. We can distinguish between self field AC losses, that is, the power losses due to transport current inside each conductor, and the magnetic AC losses due to a magnetic field external to the conductor, which we deal with in this work. The latter kind of losses are important for devices where a high magnetic field is present, like magnets and transformers.

Magnetic AC losses critically depend on the superconductor wire geometry [4,5]. As it was pointed out in [4–6], dividing the superconductor wire into filaments reduces the magnetic losses. Moreover, it is known that dividing superconducting wires into filaments and immersing them into a conducting matrix makes the wire more reliable under quenching [4,5]. In addition, it is shown that for Ag/Bi-2223 tapes, the superconducting properties improves when the superconducting region is divided into filaments with a high aspect ratio [7,8]. Then, this is the HTSC wire geometry most often met in practice.

The magnetic AC losses in multifilamentary tapes have their origin in mainly three mechanisms. They are the eddy currents in the conducting sheath, the magnetic hysteresis arising from the flux pinning in the supercon-

ductor, and the inter-filament currents (also known as coupling currents) that flow across the conducting matrix [4,5]. Although it is somehow understood how to reduce the eddy and coupling currents losses [9,10], important work remains to be done concerning the hysteresis losses. Many experimental works showed that the hysteresis losses depend strongly on the orientation of the external AC field [11–13]. It is shown that the hysteresis losses under an applied field H_a perpendicular to the wide face of the HTSC tapes are more than one order of magnitude higher than if H_a is either parallel to the wide face or in the transport direction.

Although the losses for H_a parallel to the tape wide face or in the transport direction are theoretically well described by the Bean's model for a slab [14,9,15], there is not any theoretical model that satisfactorily describe the losses of multifilamentary tapes under perpendicular H_a [16,17]. The only theories so far describe partially the magnetic properties and hysteresis losses of multifilamentary tapes, in cases such as infinite z -stacks and x -arrays of infinitely thin strips in the critical state model [18], realistic multifilamentary tapes but considering complete shielding only [19,20], and z -stacks of strips [21,22].

In the first paper of this series [21] we introduced a general model for calculating the magnetic response of a finite thickness superconductor of infinite length within the critical state model [14] applied to a vertical stack of infinitely long superconducting strips. A case most often encountered in real superconducting cables is that of an array of superconducting filaments arranged in rows and columns. The purpose of this paper is to numerically calculate and discuss the main magnetic properties of superconducting multifilamentary tapes, such as field profiles, magnetization curves, and magnetic AC losses within the critical state model in a perpendicular applied field. We consider the realistic case that arrays have a finite number of filaments, each having finite thickness. For all cases the external applied field H_a is considered to be uniform and perpendicular to the infinite dimension.

The are two cases we study separately. In the first

one, current is restricted to return through the same filament. We refer to this case as isolated filaments. This is the desired case for AC magnetic losses reduction in real HTSC tapes [4–6,23]. The other case is when current can go in one direction in a given filament and return through another one. We refer to the latter case as completely interconnected filaments. This is the limiting case of filaments with a high number of intergrowths [24–28] or when coupling currents through the conducting matrix are of the same magnitude as the superconducting currents [9,23,29,30]. As explained in these references and below, the magnetic behavior for each filament connection case is strongly different. Therefore, a detailed study of ac losses in superconducting cables should include these two cases. The strong difference in considering interconnected or isolated strips can be realized in the current and field profiles shown in Fig. 6 for horizontal arrays.

Although in the present paper only calculations for filaments with a high aspect ratio like those present in the actual tapes are shown, the numerical method presented below has been checked to be useful for any thickness-to-width ratio between 0.001 and 100.

The present paper is structured as follows. In section II we present the calculation model and its modifications from the original one in [21]. Current and field profiles are calculated and discussed in Section III. The results of magnetization and magnetic AC losses are discussed in Section IV and V, respectively. Finally, in Section VI we present the main conclusions of this work. The full penetration field for x -arrays and xz matrices can be analytically calculated, being described in the Appendix.

II. MODEL

We assume the x -arrays and xz matrices to be made up of identical rectangular strips, which we consider infinitely long in the y direction, as in [21]. The separation of the xz -matrix rows is h and the separation of columns is d (an x -array can be considered as a matrix with a single row), as shown in Fig. 1. The strips have dimensions $2a$ and $2b$ in the horizontal and vertical directions, respectively, and are divided into $2n_x \times 2n_z$ elements with cross-section $(\Delta x)(\Delta z)$. We consider that, if present, the current density that flows through each element is uniform.

As explained above, we discuss two different cases of filament connection: when the filaments are all interconnected or when they are isolated to each other. To describe both cases we use mainly the same model previously used for z -stacks [21] and for cylinders [31,32], which is based on minimizing the magnetic energy of current distribution within the critical state model. We name this model as the minimum magnetic energy variation procedure, thereafter referred to as MMEV. As in [21], we assume there is no equilibrium magnetization nor

field dependence of J_c . The method allows the calculation of $J(x, z)$ in the initial magnetization process of a zero-field cooled superconductor, from which the initial magnetization curve, the complete magnetization loop and the AC susceptibility can be easily calculated [21].

The numerical procedure explained below is valid for any superconductor wire geometry as long as it has yz mirror symmetry. So, this model can be applied to describe realistic multifilamentary tapes, and not only x -arrays and xz matrices.

We now discuss two features that are needed to apply the MMEV procedure to a certain superconductor geometry. This will help us to determine which modifications, if necessary, have to be done to adapt the MMEV procedure to x -arrays and xz matrices for the cases of interconnected and isolated strips.

The first condition to apply the MMEV procedure is that one needs to know the shape of the closed current loops of the magnetically induced current for any applied field value H_a (that is, one has to know what is the returning current element for a given one, so that both form a close current loop). For cylinders the closed current loops were simply circular [31,32], while for z -stacks [21] they were made up by an infinite straight current in the y direction centered at (x, z) and by another centered at $(-x, z)$, which formed a closed circuit at infinity. For the latter case, the sign criterion for the current flowing in a circuit is to take it positive when the current in the $x \geq 0$ region follows the positive y axis, and negative otherwise.

Another feature that has to be taken into account in order to apply the numerical method in Ref. [21], developed for z -stacks, to x -arrays and xz matrices is the sign of the induced current. For simple geometries such as rectangular strips and disks [33–37], elliptical tapes [36,38,39], and z -stacks [18,21], current in the initial magnetization curve is negative for all circuits, that is, current profiles are symmetrical to the yz plane. However, this feature it is not so obvious for geometries with gaps in the horizontal direction like x arrays and xz matrices.

As explained in the following subsections, the applicability of the mentioned two features is different when considering interconnected or isolated strips. So, the adaptation of the MMEV procedure to the two different connection cases must be considered separately.

A. Interconnected strips

For the present case of x -arrays and xz -matrices with completely interconnected strips the two features presented above are still valid. This can be justified as follows.

First, the closed circuits to be used in the simulations are the same as for z -stacks, which are each pair of current elements centered at (x, z) and $(-x, z)$, Fig. 1. This is so because of the mirror yz symmetry of the system

and the fact that the strips are interconnected at infinity so that currents belonging to different strips can be closed.

Secondly, the fact that in the initial magnetization curve the current is negative in all the circuits is also valid for the present case. We arrive to this conclusion after doing some preliminary numerical calculations, in which we changed the original numerical method letting the procedure to choose which sign in each circuit is optimum to minimize the energy. After doing so, we saw that in the initial magnetization curve and for a given H_a , current is the same and negative for all circuits, except for very few circuits on the final current profile due to numerical error. Notice that this means that the current of the strips at the $x \geq 0$ region return to those in the $x \leq 0$ region, so that current return through different strips for all circuits except for those centered at $x = 0$. This result is the expected one, because this situation is the one that minimizes the most the energy, so it should be the chosen one when there are no restrictions.

Then, we conclude that numerical method and formulae for x -arrays and xz matrices are the same as those previously used for z -stacks [21] with the only modifications needed for adapting the model to the new geometry.

B. Isolated Strips

The model used to describe current isolated strips must take into account that all real current loops have to be closed inside each strip, so that there has to be the same amount of current following the negative y direction than the positive one inside each strip. In addition, although the current distribution of the whole x -array or xz -matrix have yz mirror symmetry for the plane $x = 0$, the current distribution in the individual strips is not necessarily symmetrical to their yz central plane, except for those centered at $x = 0$.

Then, the features of the MMEV procedure described above do not apply, so that we need to do significant modifications to the original numerical procedure presented in [21].

The actual current loops in this case have the shape of two straight lines within the same strip carrying opposite currents and closed at infinity (solid lines in Fig. 2). These straight currents can be identified with the elements which the strips are divided in. The main difficulty is to know which pairs of elements describe closed current loops.

To help solving this problem we notice that, thanks to the overall mirror symmetry to the yz plane at $x = 0$, for any closed current loop in a strip at the $x \geq 0$ zone, there is another current loop set symmetrically in the corresponding strip in the $x \leq 0$ zone (Fig. 2). Furthermore, if we take as closed current loops the pairs of elements set symmetrically to the yz plane (dashed lines in Fig. 2), the total current distribution is the same

except at the ends, which do not modify the magnetic moment if we consider the strips long enough. Both systems of closed circuits have the same magnetic properties, including magnetic energy and magnetic moment. Consequently, since these symmetrical pairs of elements correspond to the closed loops used for z -stacks in [21], all the formulae presented there are still applicable.

Taking these symmetrical pairs of elements as closed loops for the numerical procedure, as done in Sec. II A for interconnected strips, and the fact that current loops must close inside each strip, the MMEV procedure for isolated strips becomes in this case:

1. For a given applied field H_a , a given current distribution, and for each pair of strips set symmetrically to the yz plane (symmetrical pair of strips), there are found: i) the loop where setting a negative current would reduce the most the magnetic energy and ii) the loop where setting a positive current would rise it the least. These loops are referred to as a pair of loops.
2. The pair of loops that lower the most the magnetic energy is selected among all those belonging to each symmetrical pair of strips.
3. A current of the corresponding sign is set in the selected loops.
4. This procedure is repeated until setting current in the most energy-reducing pair of loops would increase the energy instead of lowering it.

Notice that each pair of loops where current is set in the simulations describes two real closed current loops belonging to each strip that constitute the symmetrical pair of strips.

III. CURRENT PENETRATION AND FIELD PROFILES

For the sake of clarity, we discuss separately the results corresponding to the situation in which tapes are interconnected and that in which they are isolated.

A. Interconnected strips

We first discuss the current and field penetration profiles calculated for an x -array composed of three filaments with dimensions $b/a = 0.1$. In Fig. 3 we show the current profiles and the field lines corresponding to three x -arrays with varying separation between the individual tapes. The applied field in all cases is $0.2H_{\text{pen}}$, being H_{pen} the penetration field for the whole x -array (Appendix). The common behavior observed is that currents are induced to try to shield not only the superconductors

(field is zero in the current-free regions inside the superconductors) but also the space between them. Actually, we find that there appears an overshielding near the inner edge of the external strips (Fig. 3), so that the field there is opposite to the external field. This feature has been previously predicted for rings in the critical state [40] and for completely shielded toroids [41].

The general trends described above for the case of an x -array are also valid for the case of an xz -matrix. Actually, it is important to remark that the general trends in current and field penetration and the magnetic behavior of an xz -matrix result from the composition of the properties of both the x -array and the z vertical stack that forms it. In Fig. 4 we show the calculated current penetration profiles for an xz -matrix made of nine strips (3×3), each with dimensions $b/a = 0.1$ corresponding to an applied field of $0.2H_{\text{pen}}$. We also plot the total (left figures) and self (right) magnetic field, that is, the sum of the external magnetic field plus that created by the superconducting currents and only the latter contribution, respectively. The general trend of shielding the internal volume of the region bounded by the superconductor, including gaps between tapes, is also clearly seen. An interesting feature is that a very satisfactory magnetic shielding is achieved for the three different arrays, as illustrated from the fact that the self-field in the central region has in all cases a constant value over a very large region. However, this shielding is, for the values of the applied field considered here, basically produced by the tapes in the two outer vertical columns, which are largely penetrated by currents. Only a little current is needed to flow in the upper and bottom tapes of the inner column to create a fine adjustment of the field in the central region.

B. Isolated strips

We now present the results calculated for the case that the superconducting strips are isolated so that current has to go and return always through the same filament. We start again with the case of an x -array composed of three strips with dimensions $b/a = 0.1$. In Fig. 5 we show the current profiles and the field distribution calculated for three x -arrays with varying separation between the individual tapes. The applied field is $0.1H_{\text{pen}}$. Again, all tapes have dimensions $b/a = 0.1$. By simple inspection, one can realize how important the differences are with respect to the case of interconnected strips. In the present case of isolated strips there is appreciable current penetration in all the strips and not only in outer ones, although the magnetic coupling between them makes the current distribution in the outer strips different from the central one. Another important effect to be remarked is that there is an important flux compression in the space between the strips. Since all strips tend to shield the magnetic field in their interiors, the field in the air gap

between each pair of strips is stronger because of the field exclusion in both adjacent strips. Actually, field lines are very dense not only in the gap between strips but also in a zone in the strips nearest to the gap, where current penetrate an important distance (this effect is particularly clear for the case of the smallest separation). This compression effect was also found by Mawatari for the case of x -arrays of very thin strips [18], by Fabbriatore et al for x -arrays, xz matrices and realistic shapes of multifilamentary tapes in the Meissner state [19,20], and by Mikitik and Brandt for a completely shielded double strip [42].

We can better compare the current and field profiles for the interconnected and isolated cases by looking at figure 6, where we plot current profiles for the x -array with separation $d = 0.2a$ for both interconnected and isolated cases. It can be seen that for interconnected strips, current penetrates earlier (that is, for lower values of the applied field) in the outer tapes, since currents flowing there create an important shielding not only in each strip but in the whole space between them. On the other hand, in the isolated strips case currents returning through the same strip create a field compression in the channels (that include the gaps and a portion of each strip near the gap), so that the amount of current penetration is similar for the three strips. The current distribution when the strips are close to each other is slightly asymmetric with respect to the central plane of each strip, because the field in the channels felt by the inner sides of the outer strips has a different spatial distribution than the homogeneous applied field felt in the outer sides. We have found that this asymmetry increases for thicker filaments, that is, higher b/a (not shown).

We now present some results for the xz -matrix array for the case of isolated strips. As said above, results for the matrix can be understood from the composition of the effects of horizontal and vertical arrays. In Fig. 7 we show the calculated current penetration profiles for an xz -array made of nine strips (3×3) with dimensions $b/a = 0.1$ corresponding to an applied field of $0.1H_{\text{pen}}$, together with the total (left figures) and self (right) magnetic field. The two cases correspond to separations of $d/a = h/a = 0.02$ and 2, respectively. The effect of flux compression along the vertical channels that include the gaps and the surrounding regions is clearly seen in the case of the smallest separation distance.

For the case of xz matrices it can be observed how field is shielded in the vertical gaps between rows but it is enhanced in the horizontal gaps between columns. Then, for isolated strips, magnetic interaction between rows and columns have opposite effects. Furthermore, the difference between the field in the vertical gaps and the applied field H_a is much higher than for the horizontal gaps, as can be seen in Fig. 7 for the xz -matrix with higher separation. This implies that the magnetic coupling between strips in the horizontal direction is lost at smaller distances than that in the vertical direction.

IV. MAGNETIZATION

A. x -arrays

We now analyze the results for the magnetization of the x -arrays. In Fig. 8 we plot the calculated magnetization M as function of the applied field H_a for the 3 x -arrays of Figs. 3 and 5. For each strip $b/a = 0.1$, and the separation distance between strips is, for each case, $d/a = 0.02, 0.2$ and 2 . The upper figure shows the results for the isolated strips whereas the data in the bottom part is for interconnected strips.

The magnetization for both isolated and interconnected strips shows important differences, arising from the different current penetration profiles studied in the Section III. We first discuss the results for isolated strips. It can be seen that M saturates at smaller values than for the case of interconnected strips and that this saturation value is the same for the different separations. The results for largest separation, $d/a = 2$, are not very different from the results obtained from a single strip with $b/a = 0.1$, corresponding to the limit of complete magnetically uncoupled strips, which is also shown in the figure. An important result is that the initial slope of the $M(H_a)$ curve χ_0 increases (in absolute value) with decreasing separation. The reason for this behavior can be traced back to the presence of the flux compression effect discussed in Section IIIB, since a smaller separation means thinner channels and a corresponding larger flux compression. The enhancement of the initial slope can also be explained by the fact that the strips have to shield not only the external applied field but also the field created by the other strips. This enhancement of χ_0 have already been predicted in similar situations [18–20,26].

We have found that the initial slope calculated with our approach is coincident, within a 4% difference, with that calculated numerically by finite elements by Fabbriatore et al [19] for the case of 5×5 and 5×3 filament matrices with complete shielding. The initial slope has also been compared with other works [43,44] for a single strip for a high range of b/a ($0.001 \leq b/a \leq 100$), obtaining a difference smaller than a 1%.

When comparing the results for isolated strip to the case of interconnected ones, important differences appear. A first difference is that the saturation magnetization in the latter case is not only larger in general with respect to the isolated case but also depends on the separation. The second difference is that the trend found when decreasing the separation distance between tapes is reversed: whereas for isolated strip decreasing separation distance d/a results in a larger (in absolute value) slope of the initial magnetization, for interconnected strips the slope gets smaller with decreasing separation.

We explain the reasons for both differences as follows. The difference behavior in the saturation magnetization arises from the fact that this value corresponds to the magnetic moment per unit volume when all the strips

are fully penetrated. The magnetic moment is proportional to the area threaded by the current loops, which in interconnected case are not restricted to a single strip but they can span from even one extreme of the array to the other. Actually, the saturation magnetization M_s can be analytically calculated considering that, for isolated strips, at saturation the $\pm J_c$ interface is close to a straight line, so that M_s is the same as for a single uncoupled strip, being $M_s = 1/2 J_c a$ [14,33,34]. For the case of interconnected strips, the current distribution at saturation is $\mathbf{J} = -J_c \hat{\mathbf{y}}$ for $x \geq 0$ and $\mathbf{J} = J_c \hat{\mathbf{y}}$ for $x \leq 0$, so that M_s can be calculated as

$$M_s = \frac{J_c}{2} (2a + d) \left(\frac{n_{f,x}}{2} \right) \quad (n_{f,x} \text{ even}) \quad (1)$$

$$M_s = \frac{J_c}{2n_{f,x}} \left[a + (2a + d) \frac{(n_{f,x}^2 - 1)}{2} \right] \quad (n_{f,x} \text{ odd}), \quad (2)$$

where $n_{f,x}$ is the number of strips in the x direction for either an x -array or an xz -matrix.

As to the initial slope of the $M(H_a)$ curve, in the case of interconnected tapes the flux compression effect discussed above does not exist so the reason for the behavior of the initial slope of the $M(H_a)$ curve must be a different one. The governing effects now are the demagnetizing effects arising from the large aspect ratio of the x -array taken as a whole. The demagnetizing effects tend to enhance the initial slope [31,45,46] when the sample aspect ratio increases. Therefore, when the separation is small the array is behaving similarly to a single strip with the same thickness but three times the width, which shows less demagnetizing effect and, as a result, a smaller (in absolute value) initial slope of the magnetization.

Another feature observed in the interconnected case is the observation of a kink (change in the slope) in the magnetization curve, particularly for the cases of large separation between strips. This effect is explained as follows. Since the magnetic moment is proportional to the area enclosed by the loops, currents in the external strips contribute more to the magnetization than those in the inner ones. So, when the external strips become saturated, new current can only be induced in the central strip, having a lower contribution to the magnetization M , so that the M rate when H_a is increased is lower in magnitude; a similar effect has been predicted for rings in the critical state model [40]. In a single strip or even in the case of an x array with isolated filaments, this process is continuous, but not in the present case of interconnected strips separated a horizontal distance.

B. xz -matrices

The magnetization of xy matrices is again a combination of the effects discussed above for horizontal arrays and in the previous paper [21] for the vertical ones. In Fig. 9 the initial magnetization curve $M(H_a)$ for xz matrices with the same vertical separation is plotted, for

a vertical separation $h/a = 0.2$, and several horizontal separations d/a . The curves are qualitatively similar to those for x -arrays and the same values of d/a , so that the discussion done for x -arrays is still valid. The main difference between x -arrays and xz matrices lies in both the value of the saturation field H_s , that is, the field which M reaches its saturation value, and the magnitude of the initial slope. For the case of both isolated and interconnected xz matrices, H_s is higher than for x -arrays, while the initial slope is lower. This is due to the reduction of the demagnetizing effects owing to the stacking in the z direction [21,18,19]. Moreover, the mentioned differences of the $M(H_a)$ curve between x arrays and matrices would be qualitatively the same if we considered an x -array with a larger filament thickness. Detailed results of the magnetization of xz matrices calculated by our model will be presented elsewhere.

V. AC LOSSES

A. x -arrays

In this section we study the imaginary part of the AC susceptibility, χ'' , calculated from the magnetization loops obtained in section IV, which can be easily related to the AC losses [47]. In Fig. 10 we present calculated results for χ'' as function of the AC field amplitude H_{ac} for the same x arrays discussed in the previous sections (with $b/a = 0.1$ and different separation distances $d/a = 2, 0.2$, and 0.02). The two different cases of interconnected and isolated strips are plot together for comparison. Results show that the general trend is the appearance of a peak in the χ'' curve (and therefore a change of slope of the AC losses). This peak, however, is wider for the case of isolated strips (also shown in the figure), specially on the left part of the peak. This effect has been experimentally found in several works [16,17,48,49]. Actually, the cause of the disagreement between theoretical predictions and experiments in these works is that they used models for single strips or disks, which yielded narrow peaks. Our model allows for the explanation of this effect. Concerning hysteresis losses only, as we do in this work, the reason for this widening of the peak is that the $M(H_a)$ curve becomes non linear at small applied field values because of the penetration of magnetic flux not only in the outer surface regions of the tapes but also in the channels between strips, where the field intensity is enhanced. This deviation from linearity in the $M(H_a)$ curve results in an increase of the loss.

We also find that decreasing the distance between strips results in a higher or a smaller value of the peak, depending upon we are considering the isolated or interconnected case, respectively. This dependence on separation distance is only slight for the case of isolated strips and much more evident for the interconnected ones. These results can be understood from the magnetization

curves of Fig. 8, in which we observe two important properties: the initial slope of the magnetization curve for interconnected strips increases (in absolute value) with increasing distance between strips, while it decreases for isolated strips, and, most important, for interconnected strips, the saturation magnetization has very different values for the different separations, while it remains almost constant for isolated strips. All these effects have been explained in section IV. Another characteristics observed in the two upper curves of Fig. 10 is a kink at a particular field value, that is directly related to the presence of a similar kink in the magnetization data shown in Fig. 8. This kink was already predicted for rings [40] and later experimentally observed [50]. Furthermore, experimental evidence of a kink in actual superconducting tapes was shown for the case of a Ag/Bi-2223 tape with the superconducting core shaped as a circular shell [51] or two concentric elliptical shells [29].

Another interesting result for the χ'' calculations is shown in Fig. 11, where we show the calculated results for x -arrays of several strips with $b/a = 0.01$ with a fixed separation distance of $d/a = 0.02$. Results are shown for arrays of 2, 3, 5, and 9 strips. We consider the isolated strips case, in order to compare our results with the analytical prediction for an infinite array of Mawatari [18]. We also include the calculated result for a single strip with $b/a = 0.01$ as well as the same curve calculated from the analytical formulas for thin strips [34]. The small difference between the two latter results indicates that $b/a = 0.01$ is already a satisfactory value for using the thin strip approximation. On the other limit, we check that the results for a large number of tapes tend to the analytical results of Mawatari [18], although 9 is not a sufficient number for approaching the limiting case (higher number of tapes yield values closer to Mawatari's results; not shown for clarity). The general trend observed that the losses increase with the number of tapes is due to the fact that the effect of the channels discussed above increases for higher number of strips [19,20,26].

B. xz -matrices

In Fig. 12 we present the dependence of χ'' upon the AC applied field amplitude H_{ac} for xz matrices with $b/a = 0.1$, $h/a = 0.2$, and several values of d/a . It can be seen that the qualitative variations of the $\chi''(H_{ac})$ curve when considering isolated or interconnected strips is the same as for x arrays, as well as the effect of changing d/a . However, for xz matrices there is both a reduction of the peak in the $\chi''(H_{ac})$ curve and a shifting to higher H_{ac} values. These facts can be explained returning to the initial magnetization curves in Figs. 8 and 9, where the initial slope was lower for all xz matrices and the saturation field was higher. A detailed study of the AC losses from the χ'' values, including the real part of the susceptibility, χ' , for xz matrices will be presented

elsewhere.

VI. CONCLUSIONS

We have presented a model that allow to study the response of a horizontal array of superconducting strips of finite thickness in a perpendicular applied field. The different cases of isolated and completely interconnected strips have been discussed separately. Current penetration results show that whereas in the interconnected cases the filaments magnetically shield the whole internal volume of the tape, in the case of isolated strips, the shielding is within each of them. The latter effect in the isolated strip case creates channels of field compression between the strips, particularly when the separation distance between strips is small. These channels govern the magnetic and AC losses properties of the arrays of isolated tapes. Because of them, when decreasing the horizontal distance between strips, the initial slope of the magnetization curve increases (in absolute value), and, correspondingly, there are larger AC losses. Moreover, the experimentally found effect of a widening of the peak in the imaginary part of the AC susceptibility can be explained by the same effect. On the other hand, for the case of interconnected strips, the trend is the opposite: decreasing the horizontal distance between strips the initial slope of the magnetization curve and the AC losses are reduced. The effect governing these features are now the demagnetizing effects: when strips are close to each other they behave as a single tape with smaller aspect ratio and, therefore, with smaller demagnetizing effects.

The magnetic properties of superconductor matrix arrays are a composition of those for horizontal and vertical arrays, discussed above and in the first paper of this series, respectively. A result of practical importance is that AC losses are reduced when decreasing the vertical separation between strips in the tape, because when stacking strips in the vertical direction they behave as thicker strips and therefore have less demagnetizing effects and less AC losses.

In the present version, the model cannot be used to the study of the case in which a transport current flows in the array in addition to the applied magnetic field. This extension will be presented elsewhere.

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APPENDIX A: FIELD OF FULL PENETRATION

All previous results are calculated numerically. In this appendix we provide some analytical calculations that may be useful in the practice.

As explained in [21,52,35], the full penetration field can be calculated as minus the field created by the current distribution \mathbf{H}_J in the last induced current point, where $\mathbf{H}_J = H_J \hat{\mathbf{z}}$. Then, both the current distribution at the penetration field and the last induced current point must be known to calculate H_{pen} .

For x -arrays and xz matrices we differentiate again two cases depending on the way that the strips are connected at infinity: completely interconnected strips and current isolated strips.

1. Completely interconnected strips

For this case, the volume current density at the penetration field is $\mathbf{J} = -J_c \hat{\mathbf{y}}$ for $x > 0$ and $\mathbf{J} = J_c \hat{\mathbf{y}}$ for $x < 0$.

When both the number of strips in the x axis $n_{f,x}$ and in the z axis $n_{f,z}$ are odd, the last induced current point \mathbf{r}_m is simply the center of the central strip. Using the Biot-Savart law to calculate $H_{J,z}(r=0)$, we obtain

$$H_{\text{pen,matrix}}(n_{f,x}, n_{f,z}) = H_{\text{pen,stack}}(n_{f,z}) + \frac{J_c}{2\pi} \left[2 \sum_{i=1}^{\frac{n_{f,x}-1}{2}} F_2((2a+d)i, 0, a, b) + 4 \sum_{i=1}^{\frac{n_{f,x}-1}{2}} \sum_{j=1}^{\frac{n_{f,z}-1}{2}} F_2((2a+d)i, (2b+h)j, a, b) \right], \quad (\text{A1})$$

where $H_{\text{pen,stack}}$ is the penetration field for a z -stack [21] and the function $F_2(u, v, t, d)$ is defined as

$$F_2(u, v, t, d) = (u-t) \left[\arctan\left(\frac{v-d}{u-t}\right) - \arctan\left(\frac{v+d}{u-t}\right) \right] + (u+t) \left[\arctan\left(\frac{v+d}{u+t}\right) - \arctan\left(\frac{v-d}{u+t}\right) \right] + \frac{(u-d)}{2} \ln \left[\frac{(u-t)^2 + (v-d)^2}{(u+t)^2 + (v-d)^2} \right] + \frac{(u+d)}{2} \ln \left[\frac{(u+t)^2 + (v+d)^2}{(u-t)^2 + (v+d)^2} \right]. \quad (\text{A2})$$

The penetration field for an x -array with an odd number of strips is the same as in Eq. (A1) but removing the term with the double sum and taking $n_{f,z} = 1$.

When either $n_{f,x}$ or $n_{f,z}$ are even, the last induced current point is not easy to be determined. In those strips that current returns through the same filament, the total magnetic field increases monotonically from the edges of the strip to the current profile. When $n_{f,x}$ is odd and $n_{f,z}$ is even the last strips to be fully penetrated are those in

the central column and in the inner rows. Then, the last induced current point \mathbf{r}_m , where $H_{J,z}(\mathbf{r}_m) = -H_{\text{pen}}$, is on the z axis and can be determined as the point where $H_{J,z}$ is maximum in absolute value. When $n_{f,x}$ is even, we have found no way to analytically calculate \mathbf{r}_m and H_{pen} .

2. Current Isolated Strips

As discussed in Sec. IIIB, the current interface at the penetration field is almost a vertical straight line at the center of the strip. We have found that this approximation is reasonable even for strips with a ratio b/a as large as $b/a = 1$.

When $n_{f,z}$ is odd, the last penetrated current point is at the center of the strips belonging to the central row and the most external columns. This is so because external rows shield inner ones and external columns increase the field on the inner ones. Then, using the Biot-Savart law and assuming straight current interfaces, the penetration field for a xz -matrix with odd $n_{f,z}$ is

$$H_{\text{pen,matrix}}(n_{f,x}, n_{f,z}) = \frac{-J_c}{2\pi} \left[\sum_{i=0}^{n_{f,x}-1} F_3((2a+d)i, 0, a, b) + \sum_{i=0}^{n_{f,x}-1} \sum_{j=1}^{\frac{n_{f,z}-1}{2}} F_3((2a+d)i, (2b+h)j, a, b) \right], \quad (\text{A3})$$

where the function $F_3(u, v, t, d)$ is defined as $F_3(u, v, t, d) = F_2(u-t/2, v, t/2, d) - F_2(u+t/2, v, t/2, d)$. Notice that Eq. (A3) is valid when $n_{f,x}$ is either odd or even, while Eq.(A1) is only valid for an odd $n_{f,x}$. The penetration field for an x -array is the same as described in Eq. (A3) but removing the term with the double sum.

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FIG. 1. Sketch of the array of superconducting tapes. A xz -matrix is drawn, although all the parameters described are also valid for x -arrays. The y axis is perpendicular to the plane and it is oriented inwards.

FIG. 2. Sketch of the real closed current loops (solid thick lines) and those used in the simulation (dashed thick lines). The case of an x -array with two strips is drawn for simplicity. Four current elements are represented as elongated thin rectangular prisms where a single straight current flows following the y axis.

FIG. 3. Total magnetic flux lines and current profiles for interconnected x -arrays at an applied field of $H_a = 0.2H_{\text{pen}}$, being H_{pen} the complete penetration field for the whole x -array. The strips in the arrays have an aspect ratio $b/a = 0.1$ and the distances between strips are: (a) $d/a = 0.02$, (b) $d/a = 0.2$, and (c) $d/a = 2$. The horizontal scale has been contracted for clarity, while the vertical scale is the same for all figures.

FIG. 4. Total (left) and self (right) magnetic field lines and current profiles for interconnected xz matrices at an applied field of $H_a = 0.2H_{\text{pen}}$. For the strips $b/a = 0.1$ and $d/a = h/a = 0.02$ (a,b), 0.2 (c,d), and 2 (e,f). Vertical and horizontal scales are rescaled for clarity.

FIG. 5. The same as Fig. 3 but with isolated strips. The applied field is $H_a = 0.1H_{\text{pen}}$ and the strips have dimensions $b/a = 0.1$ spaced a distance: (a) $d/a = 0.02$, (b) $d/a = 0.2$, and (c) $d/a = 2$. The horizontal scale has been contracted for clarity.

FIG. 6. Current profiles for x -arrays with $b/a = 0.1$ and $d/a = 0.2$ for (a) interconnected strips and (b) isolated strips. The vertical axis has been expanded for clarity. The applied field values corresponding to each current profile are $H_a = 0.1, 0.2, 0.4, 0.6, 0.8$ and 1 in units of the penetration field H_{pen} for each case.

FIG. 7. Total (left) and self (right) magnetic field lines and current profiles for isolated xz matrices at an applied field $H_a = 0.1H_{\text{pen}}$. For the strips $b/a = 0.1$ and $d/a = h/a = 0.02$ (a,b), and 2 (c,d).

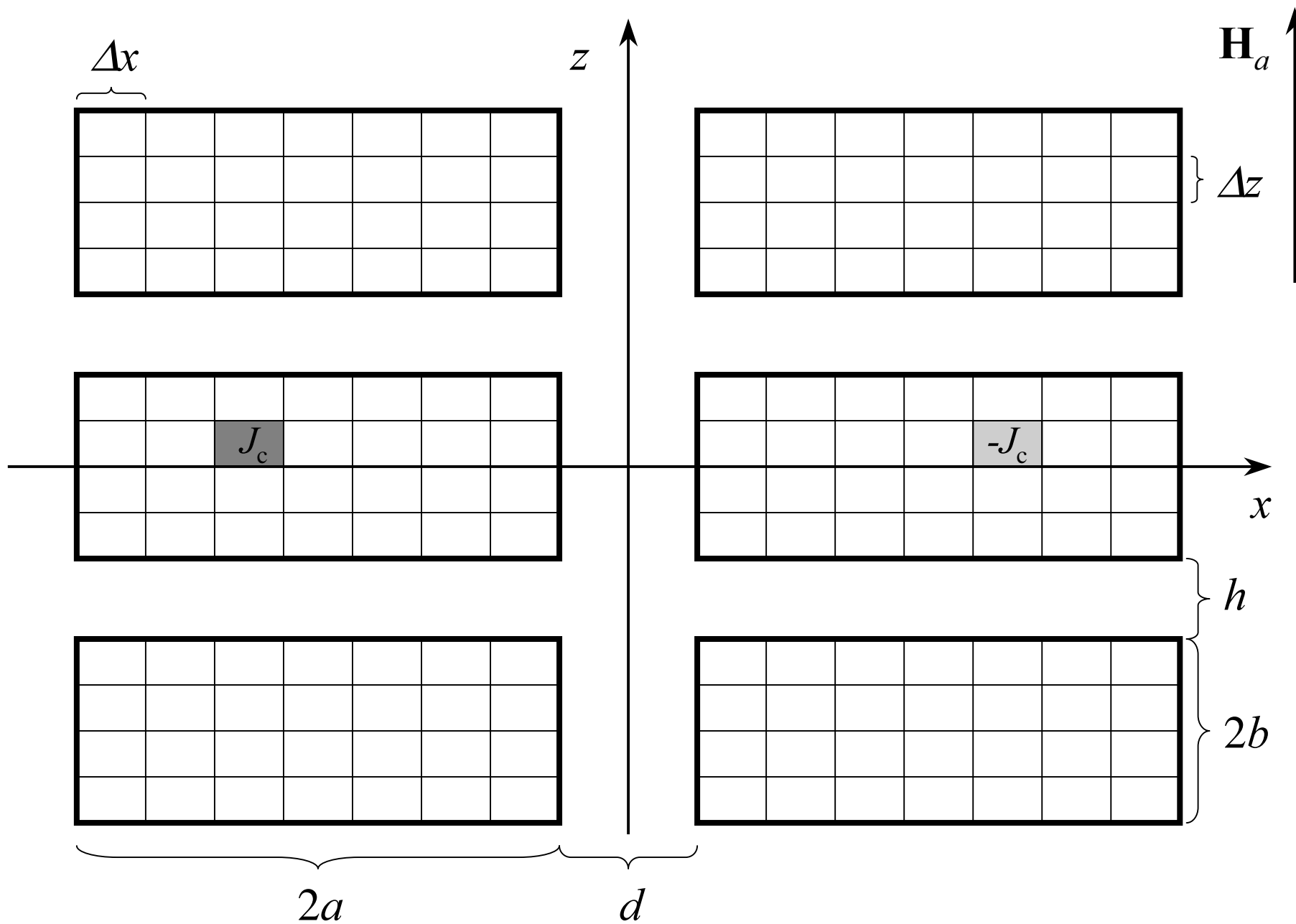
FIG. 8. Initial magnetization curves $M(H_a)$ for x -arrays with three strips with $b/a = 0.1$ and several strip separations d/a for the cases of (a) isolated strips and (b) interconnected strips. For graph (a) solid lines correspond to x -arrays with $d/a = 2, 0.2$, and 0.02 from top to bottom, while the dashed line represents $M(H_a)$ for a single strip with $b/a = 0.1$. For graph (b) solid lines correspond to x -arrays with $d/a = 0.02, 0.2$, and 2 from top to bottom and the dashed line is for a single strip with halfwidth $a' = 3a$ and $b = 0.1$.

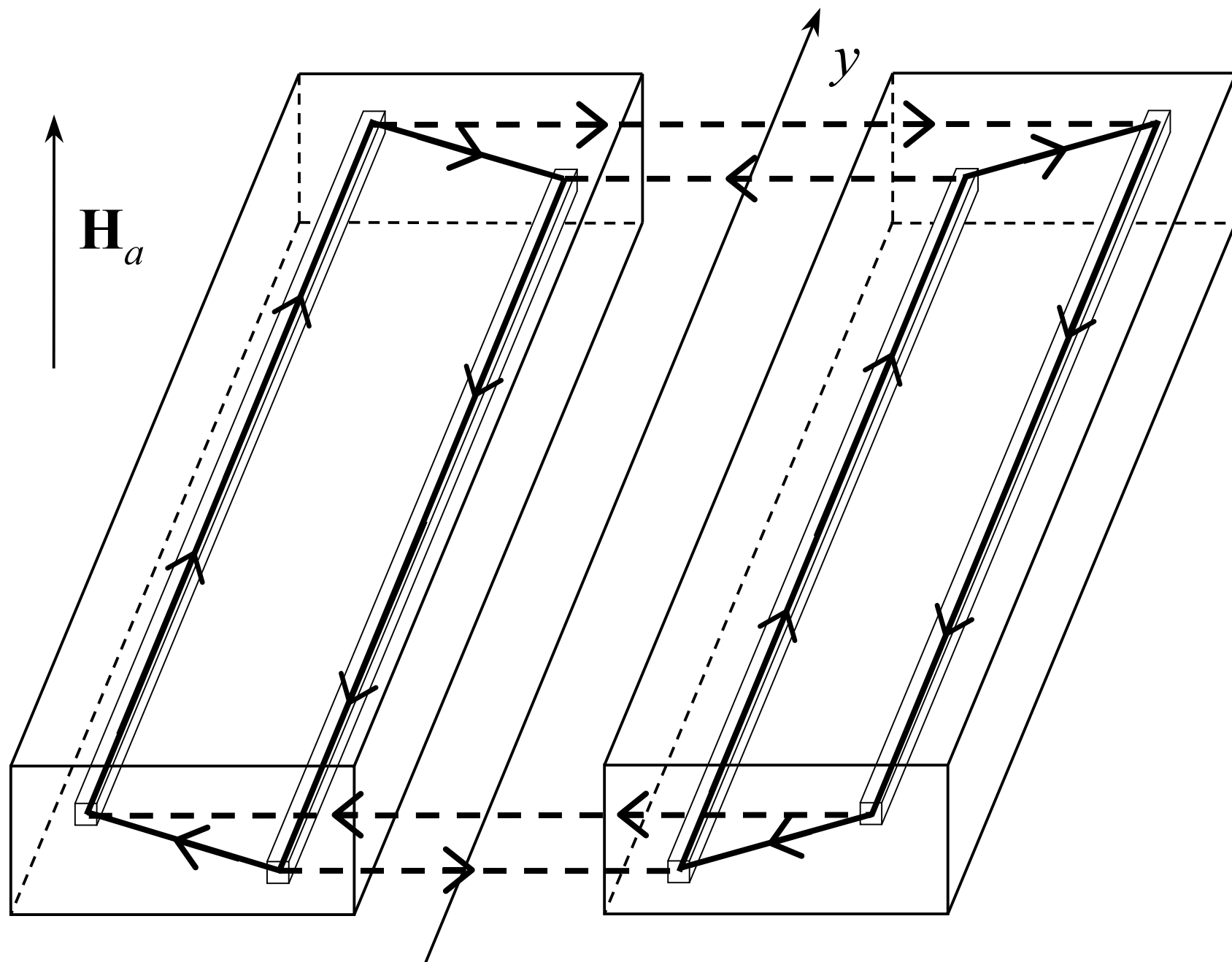
FIG. 9. Initial magnetization curves $M(H_a)$ for xz -matrices with 3×3 strips of dimensions $b/a = 0.1$, for $h/a = 0.2$ and several values of d/a for the cases of (a) isolated strips and (b) interconnected strips. For graph (a) curves correspond to $d/a = 2, 0.2$, and 0.02 from top to bottom. For graph (b) curves correspond to $d/a = 0.02, 0.2$, and 2 from top to bottom.

FIG. 10. Imaginary AC susceptibility χ'' as a function of the AC applied field amplitude H_{ac} corresponding to the $M(H_a)$ curves showed in Fig. 8 for x -arrays. The strips dimensions are $b/a = 0.1$. Solid lines are for the case of interconnected strips for $d/a = 2, 0.2$, and 0.02 from top to bottom, while dashed lines are for isolated strips with $d/a = 0.02, 0.2$, and 2 from top to bottom.

FIG. 11. Imaginary AC susceptibility χ'' as a function of H_{ac} for x -arrays with several numbers of strips n_f (solid lines), corresponding to $n_f = 9, 5, 3, 2$ from top to bottom, compared to a single strip (dashed line) and the analytical limits for a single thin strip (lower dotted line) and an infinite x array of thin strips (upper dotted line). The strips dimensions are $b/a = 0.01$.

FIG. 12. Imaginary AC susceptibility χ'' as a function of H_{ac} corresponding to the $M(H_a)$ curves showed in Fig. 9 for xz matrices. The strips dimensions are $b/a = 0.1$ and the vertical separation is fixed, being $h/a = 0.2$. Solid lines are for the case of interconnected strips for $d/a = 2, 0.2$, and 0.02 from top to bottom, while dashed lines are for isolated strips with $d/a = 0.02, 0.2$, and 2 from top to bottom.





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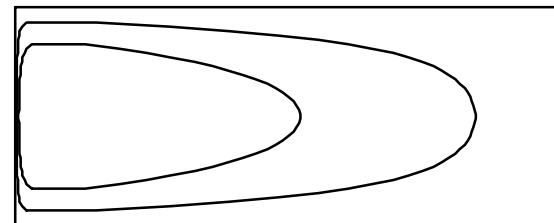
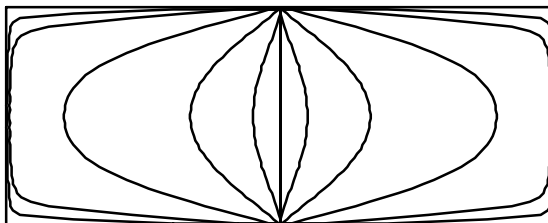
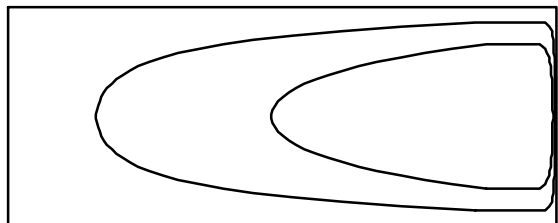
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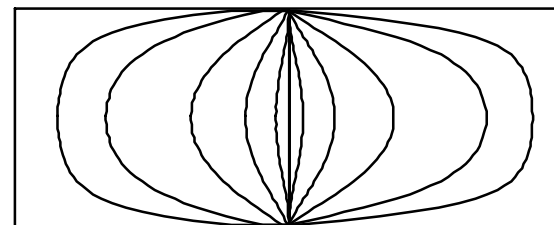
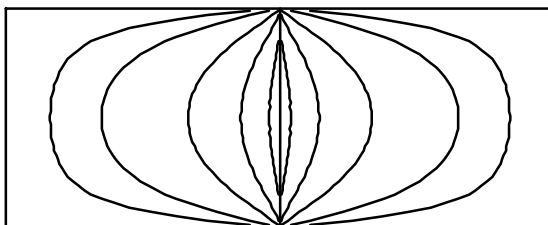
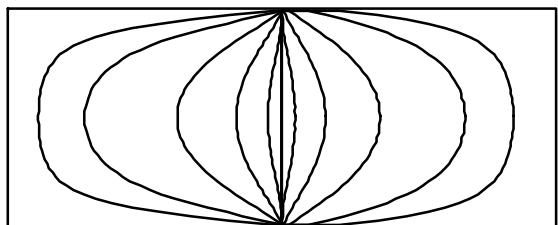
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(a)



(b)



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